Lagrangian Transport And Mixing In Geophysical Flows: A Dynamical Systems Approach

Stephen Wiggins
Control and Dynamical Systems, 107-81
California Institute of Technology
Pasadena, California 91125
Phone: (626) 395-4127 fax: (626) 796-8914 email: wiggins@cds.caltech.edu

Award #: N00014-97-1-0071
http://transport.cds.caltech.edu

LONG-TERM GOALS

To develop analytical and computational methods for the study of transport in three-dimensional, unsteady flows that is obtained as data sets through high-resolution numerical computations or remote sensing. We will develop a unified and comprehensive software package that can be used by oceanographers for such analyses.

OBJECTIVES

To develop analytical and computational methods for studying transport in several flows obtained from remote sensing data and high-resolution numerical models. In particular, we will analyze the transport properties of the flow in Monterey Bay obtained through high frequency (HF) radar measurements. Further, we will analyze transport features of the flow obtained from a high-resolution numerical model (MICOM model) of the Caribbean, Gulf of Mexico, and the Atlantic Ocean.

APPROACH

Our approach is that of the framework of nonlinear dynamical systems theory. This approach seems ideally suited for the study of transport, as there is a remarkable similarity between the mathematical framework of dynamical systems theory and the experimental and observational framework of modern oceanography. On the one hand, quasi-Lagrangian current following floats and drifters, as well as remote sensing data, show numerous localized, coherent motions ranging from major currents like the Gulf Stream to mesoscale phenomena such as rings, and associated vortex structures, down to a variety of submesoscale vortical motions. On the other hand, the theoretical tools of dynamical systems address the role that localized structures play in governing the motion over extended regions of space. Furthermore, application of dynamical systems theory relies on the existence of certain geometrical structures in the flow, not on a specific analytical form for the velocity field. Consequently, it is an ideal tool for "mining" the large data sets that result from remote sensing or large scale, high-resolution numerical simulations.

At Caltech, postdocs Chad Coulliette and Marcel Clerc are involved in developing the analytical and computational techniques. Francois Lekien is a graduate student at Caltech working in the same areas. Denny Kirwan , Bruce Lipphardt (University of Delaware), and Chet Grosch (Old Dominion University) produce the velocity fields that we analyze from the high frequency radar array operated

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **SEP 2000**	2. REPORT TYPE	3. DATES COVERED **00-00-2000 to 00-00-2000**

4. TITLE AND SUBTITLE
Lagrangian Transport And Mixing In Geophysical Flows: A Dynamical Systems Approach

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Control and Dynamical Systems, 107-81,,California Institute of Technology,,Pasadena,CA,91125

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR’S ACRONYM(S)

11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

by Jeff Paduan (Naval Postgraduate School). Kayo Ide (UCLA) works with us on the theoretical and computational aspects of the dynamical systems approach to transport theory. We are also beginning a collaboration with Annalisa Griffa and Arthur Mariano at the University of Miami.

WORK COMPLETED

We have continued our work on the analytical and computational aspects of dynamical systems theory necessary for the study of Lagrangian transport is described below.

We have developed analytical and numerical methods for finding hyperbolic trajectories in time-dependent vector fields. Such trajectories are the basic building blocks for the dynamical systems approach to Lagrangian transport.

While the notion of a hyperbolic trajectory is central to dynamical systems theory, much of the theoretical developments proceed under the assumption that a hyperbolic trajectory exists. The main goal of our work was to develop methods for finding hyperbolic trajectories in time-dependent vector fields. This brings in new mathematical issues that must be addressed in order for the dynamical systems approach to be applicable to these problems. In particular, the vector fields (or velocity fields) are time dependent and they may only be realized as data sets over finite time intervals, where space and time are discretized. Moreover, they may have a time dependence that is more complicated than the traditional periodic or quasiperiodic time dependence that is studied in dynamical systems theory. In general such flows contain an uncountable infinity of hyperbolic trajectories. We address this issue by developing the notion of a distinguished hyperbolic trajectory (DHT), which plays a central role in our work. We develop criteria for the existence of DHT's in nonlinear one and two dimensional velocity fields based on structures in the frozen time vector field. These structures are instantaneous stagnation points (SP's) and zero velocity curves. This is a very useful criterion because if the velocity field arises from data one can only view the velocity field in frozen time steps. We then develop a quantitative theory for DHT's in inhomogeneous (or “forced”) linear systems. This yields an analytical formula for the DHT for such systems. The linear theory of DHT's is central to the numerical method that we present. This numerical method can be applied to either flows that are given by an analytical velocity field or flows that are given by a data set.

Work continues on the software package ``MANGEN''. MANGEN is a new software package, which allows users to define a dynamical system with a 2D+1 discrete vector field, e.g. temporally varying 2D spatial field. MANGEN makes recent advances in dynamical systems theory and related computation methods accessible to a wide class of researchers interested in considering a given data set as a dynamical system in order obtain a more complete understanding of the nonlinearities contained within a given vector field of interest. Many concepts in dynamical systems theory are defined only in the case of infinite time and thus an unlimited amount of data. When studying a set of known differential equations, infinite time definitions and concepts are suitable. This is rarely the case in practical situations. MANGEN allows the user to define a given data set as a dynamical system and then proceed to use recently developed tools of dynamical systems to compute patchiness, locate all instantaneous stagnation points (SP’s), locate distinguished hyperbolic trajectories (DHT’s) corresponding to the SP’s, compute invariant unstable or stable manifolds of the DHT’s, locate primary intersection points, identify lobes, match lobes from time slice to time slice, and quantify flux from the lobes. From this dynamical systems analysis of the vector field, much useful information can be gain. For example, in the case of transport studies, evolving barriers and alleyways can be identified, which allows the user to better understand the nonlinearities of the data.

Using our methods, we analyzed the flow obtained from a three-layer, eddy-resolving quasigeostrophic ocean circulation model subject to an applied wind stress curl. For this model we considered transport between the northern and southern gyres separated by an eastward jet. We used techniques from dynamical systems theory, particularly lobe dynamics, for describing and quantifying geometric structures that govern transport. By "govern", we mean they can be used to compute Lagrangian transport quantities, such as the flux across the jet. We considered periodic, quasiperiodic, and chaotic velocity fields, and thus assessed the effectiveness of dynamical systems techniques in flows with progressively more spatio-temporal complexity. The numerical methods necessary to implement the dynamical systems techniques and the significance of lobe dynamics as a signature of specific ``events", such as rings pinching off from a meandering jet, was also further developed.

We have studied the surface velocity of Monterey Bay obtained from three HF radar antennae at Santa Cruz, Moss Landing and Point Pinos. The surface currents of Monterey Bay are obtained at spatial intervals of approximately 2 km and temporal intervals of approximately 2 hours by interpreting the resonant backscatter in the spectral returns for transmitted frequencies from the HF radar antenna. We have demonstrated how dynamical systems theory applies to this data and gives new insight into transport and predictability of transport processes in a coastal system. In particular, transport alleyways and fluxes can be computed exactly. There are many potential applications of this that we are now exploring

RESULTS

The main innovation of our numerical method for locating DHT's is that it provides an approximation to the DHT for the entire length (in time) of the data set. This is opposed to methods that require certain regions to converge (in the appropriate direction of time) to the DHT. The process of convergence causes one to lose much of the velocity field at the beginning and end of the time interval. Our method overcomes this problem and therefore makes dynamical systems method directly applicable to velocity fields obtained as data sets.

All algorithms for the software package MANGEN have been developed and coded. We are presently working with programmers at AMTEC to develop a graphical user interface.

In our study of transport in the wind driven double-gyre we showed that it is possible to quantify Lagrangian transport for periodic, quasiperiodic and chaotic time dependence using lobe dynamics with numerically generated velocity fields. In particular, we determined that the intergyre flux is proportional to the flow rate of the jet, the intergyre flux and the flow rate of the jet are linear functions of the wind magnitude for time-periodic flows, a catastrophic change occurs in the flux-wind relationship during the transition from periodic to chaotic time dependence, and the intergyre flux and the flow rate of the jet are nonlinear functions of the wind magnitude for chaotic flows. Concerning flow structures and the use of invariant manifolds and lobes in describing their influence on transport, we made the following conclusions. It is possible to precisely study the geometric structure of transported fluid using lobe dynamics with numerically generated velocity fields. Ring formation from a meandering jet is evident in periodic, quasiperiodic and chaotic flows. Ring formation does not contribute significantly to intergyre flux in periodic flows. Ring formation does contribute to intergyre flux in chaotic flows. All of the fluid that crosses the jet must pass through the western boundary current. Fluid that makes a transition around both gyres must follow the "figure eight" pattern dictated by the lobes.

We now describe some of our results of our dynamical systems analysis of transport in Monterey Bay using velocity fields obtained from HF radar measurements. DHTs are key Lagrangian flow features that can be viewed as the time dependent Lagrangian analog of a saddle--type separation point in the fluid. By analyzing the velocity field we are able to deduce the existence of a DHT that moves in the vicinity of Santa Cruz (shown as a red circle near the top of each panel in Figure 1) and a DHT that moves in the vicinity of Point Pinos (shown as a blue circle near the bottom of each panel in Figure 1). From dynamical systems theory we know that these DHTs possess invariant stable and unstable manifolds, and that these manifolds form the geometrical template for describing and understanding transport processes. The red and blue curves in Figure 1 show the unstable and stable manifolds associated with the DHT near Point Pinos and the DHT near Santa Cruz, respectively. Since invariant manifolds are material surfaces, in the absence of molecular diffusion they are impenetrable boundaries for fluid parcel trajectories. In Figure 1 six frames from the parcel evolution in Figure 1 are shown, with the unstable manifold (associated with the DHT near Santa Cruz) and the stable manifold (associated with the DHT near Point Pinos) both overlaid. By noting the shape and location of the manifolds (which evolve in time), we can now understand the dramatic difference in the evolution of the two fluid parcels discussed earlier. Figure 1 shows that the first parcel is released just to the right of the stable manifold, and the second parcel is released just to its left, so we know that the first parcel will leave the bay as it reaches Point Pinos, whereas the second parcel will turn eastward and reenter the bay once it reaches the DHT near Point Pinos. The stable (blue) and unstable (red) manifolds of the DHTs near Point Pinos and Santa Cruz, respectively, form a time varying alleyway that governs transport processes in the bay (Figure 1All of the transport into and out of the bay must occur near Point Pinos, where the transport alleyway opens to the west. The persistence of the unstable manifold prevents transport into or out of the bay near Santa Cruz during this time period. Understanding the transport associated with surface currents in Monterey Bay, then, comes from an understanding of the geometry of the stable and unstable manifolds.

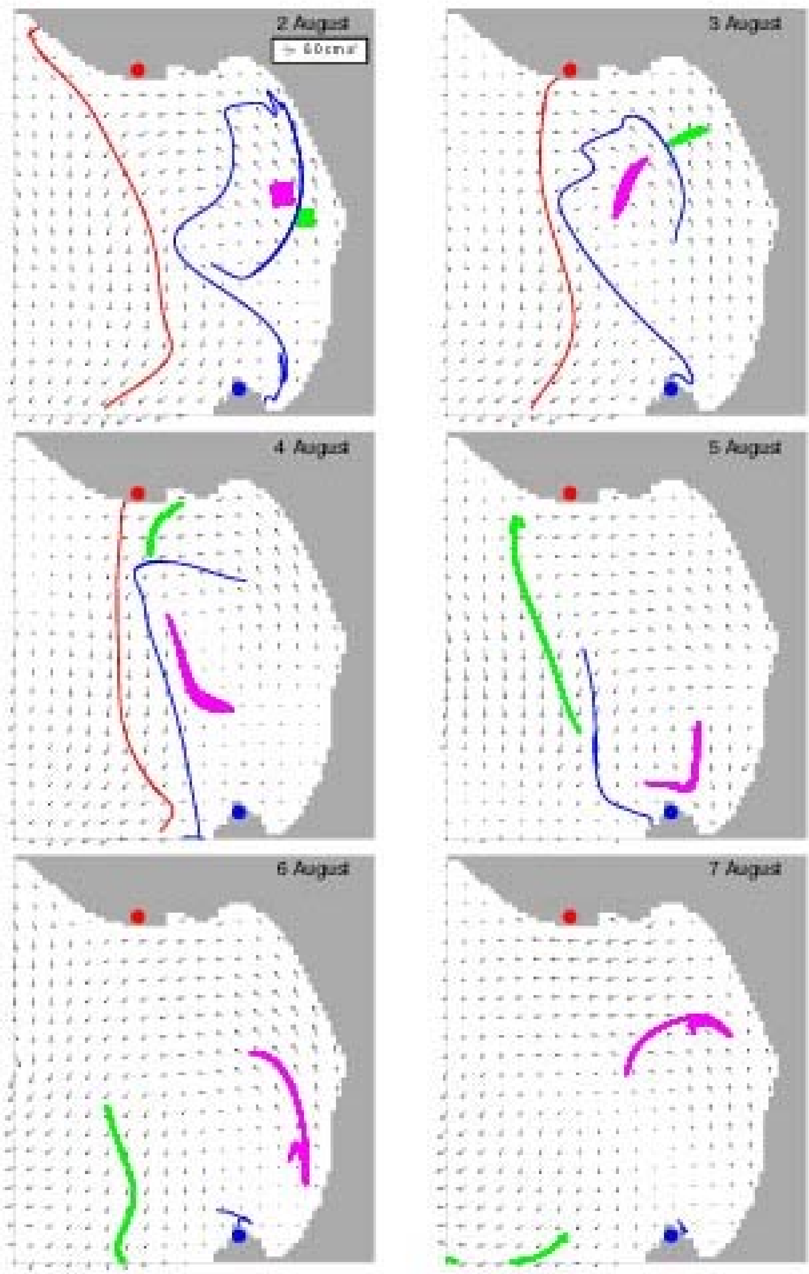

Figure 1. The influence of the stable manifold of the DHT near Point Pinos and the unstable manifold of the DHT near Santa Cruz on the evolution of two fluid parcels. Each image was made at 0430 UT on the date shown at the upper right in each panel.

IMPACT/APPLICATIONS

Our work is providing a unique set of tools for examining transport processes in velocity fields defined as data sets. In particular, we are developing analytical and computational methods to analyze transport in velocity fields obtained through remote sensing such as HF radar arrays. To our knowledge, ours are the only methods that can be used for such detailed analyses, especially for flows

that contain “organized structures”. As remote sensing technology improves, as well as our ability to handle large data sets, we expect the need and uses of our methods to further increase.

TRANSITIONS

Our analytical and computational method have been used to understand the transport properties in Monterey Bay by analyzing velocity fields obtained through HF radar measurements of Jeff Paduan (Naval Postgraduate School).

PUBLICATIONS

Wiggins, S. [1999] Chaos in the dynamics generated by sequences of maps,with applications to chaotic advection in flows with aperiodic time dependence. Z. angew. Math. Phys., 50, 585-616.

Coulliette, C., Wiggins, S. [2000] Intergyre transport in a wind-driven, quasigeostrophic double gyre: An application of lobe dynamics Nonlinear Processes in Geophysics, 7, 59-85.

Coulliette, C., Lipphardt, Jr., B. L., Wiggins, S., Grosch, C.E., Kirwan, Jr., A.D., Hatfield, J.W., Paduan, J. D. [2000] Advective transport near the coast from high frequency radar: a dynamical systems perspective. Submitted to Science.

Ide, K., Lekien, F., Wiggins, S. [2000] Distinguished hyperbolic trajectories in time-dependent fluid flows: analytical and computational approach for velocity fields defined as data sets. Submitted to Nonlinear Processes in Geophysics.

Ide, K., Wiggins, S. [2000] Eulerian transport in two-dimensional unsteady flows with a geophysical application. Submitted to Physica D.

Horner, M., Metcalfe, G., Wiggins, S., Ottino, J. [2000] Transport mechanisms in open cavities: effects of transient and periodic boundary flows. Submitted to the Journal of Fluid Mechanics.